- 1) Sketch an outline of each of the following proof methods. (10 points each)
  - a. Proof of an equality
  - b. Proof of an implication
  - c. Proof of a subset
  - d. Proof of an if-and-only-if.
  - e. Proof of an existential
  - f. Proof of a universal
  - g. Proof by cases
- 2) Explain what the following means.  $\forall_{\varepsilon>0} \exists_{N \in \mathbb{Z}_{\geq 0}} (n \ge N \Rightarrow |a_n| < \varepsilon)$  (15 points)

3) Write the following statement in mathematical notation: "There is a number whose square is smaller than any real number" (15 points)

4) Let  $f_n(x) = x^n$ . Show that for all z > 0, there is an  $n \in \mathbb{R}$  such that  $f_n(2) < z$  (100 points)